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# Wage discrimination and population composition in the long run

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## Abstract

We derive the conditions that sign the effects of changing population composition on wage levels and ratios, when labor supply and discrimination preferences vary. The overall effect depends on an aggregate market, a relative market, and a preference distribution effect.

Keywords: population composition, wage discrimination.

JEL codes: J21, J71

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## 1. Introduction

In the presence of labor market discrimination, wage differentials are a function of population composition. With employer discrimination Becker (1971a) showed that as the supply of the discriminated group increases, its wage falls and the wage of the non-discriminated group increases, resulting in a larger relative wage differential. Becker's analysis relied on perfectly inelastic labor supplies and constant discrimination preferences. In the long run labor supplies may not be fixed and discrimination preferences may change in response to a changing population composition. How does changing population composition affect wages of discriminated and non-discriminated groups when labor supplies and preferences vary? This question bears directly on any analysis of the effects of the changing racial and ethnic composition in the US in the past 30 years which saw the proportion of non-Hispanic white population falling from 80 percent in 1980 to 64 percent in 2010 (US Census Bureau, 1995, 2011). This paper shows that a changing population composition has three distinct effects on wages and derives the conditions determining the direction of each effect.

Initially Becker's taste-based discrimination models were viewed as plausible only in the short run. Subsequent theoretical work by Goldberg (1982) and Charles and Guryan (2007) showed how employer discrimination may endure in the long run and Charles and Guryan (2008) found empirical support for Becker's model for the period 1972-2004.

In this paper we follow Goldberg's (1982) articulation of Becker's (1971a) employer discrimination model and allow for labor supply to vary at the extensive margin and preferences to change with population composition. The focus on the extensive margin is empirically relevant and theoretically convenient. Unlike its effect

on hours, the effect of wages on participation can be assumed monotonic. Moreover it is widely accepted that the labor supply responsiveness at the extensive margin dominates that of the intensive margin (Heckman 1993).

## 2. The model

Consider two types of workers,  $M$  and  $F$ , with identical productive capacity. Employers dislike employing workers of type  $F$ , with this distaste expressed in a discrimination coefficient  $d_F$ . When the market wage for the  $F$  workers is  $w_F$ , employers value it as  $(1 + d_F)w_F$  with  $d_F \geq 0$ . Following Becker (1971b), employers' preferences are expressed as:

$$U = \Pi - d_F w_F L_F \quad (1)$$

where  $\Pi$  denotes profits and  $L_F$  the employment of  $F$  workers. An employer's problem is to maximize utility subject to:

$$Q = f(L) = f(L_M + L_F) \quad (2)$$

$$\Pi = Q - (w_F L_F + w_M L_M) \quad (3)$$

where the price of output is taken as the numeraire and  $f' > 0$ ,  $f'' < 0$ . From the first order conditions we have:

$$\begin{aligned} f' &= w_M && \text{if } L_M > 0, \text{ and} \\ f' &< w_M && \text{if } L_M = 0 \end{aligned} \quad (4)$$

$$\begin{aligned}
f' &= w_F(1 + d_F) && \text{if } L_F > 0, \text{ and} \\
f' &< w_F(1 + d_F) && \text{if } L_F = 0
\end{aligned} \tag{5}$$

Conditions (4) and (5) imply that in a competitive labor market with a continuous distribution of  $d_F$  across employers and for given wages for  $M$  and  $F$ , a firm hires either only  $M$  or only  $F$  workers. If an employer's  $d_F$  is such that  $w_M > w_F(1 + d_F)$ , the relative market wage differential between  $M$  and  $F$  is higher than this employer's discrimination coefficient, and therefore only  $F$  are employed. For such a firm the marginal cost of hiring  $F$  workers is always below the marginal cost of hiring  $M$  workers. Similarly, if an employer's  $d_F$  is such that  $w_M < w_F(1 + d_F)$ , only  $M$  workers are hired.

If  $d_F$  has a density  $h(d_F; p_M)$ , then  $x = \frac{1}{d_F + 1}$  has a density  $g(x; p_M)$  which, in principle, can be derived from  $h(d_F; p_M)$  (see Goldberg (1982)). The distribution of discrimination coefficients depends on the parameter  $p_M$ , the proportion of the non-discriminated group. We have no strong priors on how population composition affects discriminatory preferences. In the sociology and psychology literature the inter-group threat theory suggests that discrimination increases as the proportion of the discriminated group increases while the inter-group contact theory points to conditions that generate the opposite effect (Dixon, 2006, Pettigrew 1998). In terms of  $x = \frac{1}{d_F + 1}$ , inter-group threat theory implies that the distribution of  $x$  for higher  $p_M$  first-order stochastically dominates that for lower  $p_M$ . Inter-group contact theory implies the reverse.

Individuals either work for a fixed number of hours or not at all. If the cumulative distribution function of reservation wages of group  $k$  is given by  $S_k(w_k)$ ,  $k = F, M$ , then  $S_k(w_k)$  is the employment rate of group  $k$  at wage  $w_k$ . The equilibrium wages of groups  $F, M$  are determined by:

$$p_M S_M(w_M) = \int_0^{w_F/w_M} R(w_M) g(x; p_M) dx \quad (6)$$

$$p_F S_F(w_F) = \int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g(x; p_M) dx \quad (7)$$

where  $p_k$  is the population proportion of group  $k = F, M$ ,  $R(\cdot) = [f']^{-1}(\cdot)$  is a firm's labor demand, and population size is normalised to one.

Equations (6) and (7) indicate that the  $F/M$  wage ratio regulates the clearing of the markets for each group. In equilibrium the aggregate supply of  $M$  workers equals the sum of the demands of those firms with  $x < \frac{w_F}{w_M}$ . The aggregate supply of the  $F$  workers equals the sum of the demands of those firms with  $x > \frac{w_F}{w_M}$ . The general economic problem is the simultaneous clearing of the markets for two inputs which are imperfect substitutes, with the degree of substitutability variable and endogenous at the firm level.

Goldberg (1982) discusses how firm size varies with (constant) discrimination preferences, and Becker (1971a) analyzes how shifts of the perfectly inelastic supplies of the two groups affect equilibrium wages.

### 3. Analysis

To analyze the effects of a changing population composition on equilibrium wages,

we derive  $\frac{dw_M}{dp_M}$  and  $\frac{dw_F}{dp_M}$  from equilibrium equations (6) and (7). We obtain:

$$\frac{dw_F}{dp_M} = \Delta^{-1} \cdot [E_3 \frac{w_F}{w_M^2} (S_M - S_F) + S_F (E_1 - p_M S'_M) - E_3 \frac{w_F}{w_M^2} D_1 + (E_1 - p_M S'_M - E_3 \frac{w_F}{w_M^2}) D_2] \quad (8.1)$$

$$\frac{dw_M}{dp_M} = \Delta^{-1} \cdot [\frac{E_3}{w_M} (S_M - S_F) - S_M (E_2 - p_F S'_F) + (E_2 - p_F S'_F - \frac{E_3}{w_M}) D_1 - \frac{E_3}{w_M} D_2] \quad (8.2)$$

where

$$E_1 = \frac{\partial}{\partial w_M} [ \int_0^{w_F/w_M} R(w_M) g(x; p_M) dx ] = R'(w_M) \cdot \int_0^{w_F/w_M} g(x; p_M) dx < 0$$

$$E_2 = \frac{\partial}{\partial w_F} [ \int_{w_F/w_M}^1 R(\frac{w_F}{x}) \frac{1}{x} g(x; p_M) dx ] = \int_{w_F/w_M}^1 R'(\frac{w_F}{x}) \frac{1}{x} g(x; p_M) dx < 0$$

$$E_3 = R(w_M) g(\frac{w_F}{w_M}; p_M) > 0$$

$$D_1 = R(w_M) \int_0^{w_F/w_M} g_M(x; p_M) dx = \frac{\partial}{\partial p_M} [ \int_0^{w_F/w_M} R(w_M) g(x; p_M) dx ]$$

$$D_2 = \int_{w_F/w_M}^1 R(\frac{w_F}{x}) g_M(x; p_M) dx = \frac{\partial}{\partial p_M} [ \int_{w_F/w_M}^1 R(\frac{w_F}{x}) g(x; p_M) dx ]$$

$$\Delta = -(E_1 - p_M S'_M)(E_2 - p_F S'_F - \frac{E_3}{w_M}) + (E_2 - p_F S'_F) E_3 \frac{w_F}{w_M^2} < 0$$

$$p_F = 1 - p_M$$

$E_1$  ( $E_2$ ) is the rate of change in the demand for  $M$  ( $F$ ) workers as their wage changes, keeping the equilibrium wage ratio — and therefore the density mass of  $M$  employers— fixed. These are negative as long as  $R'(\cdot) < 0$ .  $\Delta$  is signed using  $R'(\cdot) < 0$  and  $S'(\cdot) \geq 0$ .

$D_1$  ( $D_2$ ) is the effect of the changing population composition on the labor demand of the  $M$  ( $F$ ) workers, through its effect on the distribution of preferences, keeping the equilibrium wage ratio fixed. The signs of the  $D$  terms depend on the way the preference distribution changes as  $p_M$  changes and are always opposite. If  $d'_F(p_M) \leq 0$  for all  $p_M$ , then because  $g(x; p'_M)$  first-order stochastically dominates  $g(x; p''_M)$  for  $p' \geq p''$ , we have  $D_1 \leq 0$  and  $D_2 \geq 0$ . If on the other hand  $d'_F(p_M) \geq 0$  for all  $p_M$ , then the signs of  $D_1$  and  $D_2$  are reversed.

Equations (8.1) and (8.2) show that a changing population composition has three effects on wage levels.

*The aggregate market effect:*

The sign of the aggregate market effect depends on the difference in employment rates between the two groups,  $S_M - S_F$ . For example, if  $S_M \geq S_F$  and  $p_M$  increases, aggregate labor supply increases, putting downward pressure on the wages of both

groups, and the aggregate market effect on both  $\frac{dw_M}{dp_M}$  and  $\frac{dw_F}{dp_M}$  is negative.

*The relative market effect:*

The sign of the relative market effect is always negative for the group whose population proportion increases and is the effect analyzed by Becker (1971a).

*The preference distribution effect:*



This is the sum of the terms involving  $D_1$  and  $D_2$  and its direction cannot be determined a priori.  $D_1$  and  $D_2$  have opposite signs and which of the two is positive depends on whether  $d'_F(p_M) \leq 0$  or  $d'_F(p_M) \geq 0$ .

The direction of the overall effect on wage levels therefore depends on the signs and relative magnitudes of these three effects. If we have sufficient information to sign each of these effects, and if they all have the same direction, then we can predict the direction of the effect on wages. But it may be that some of these effects have opposite signs. For example, if  $S_F \geq S_M$  and  $p_M$  increases, then aggregate and relative market effects work in the same direction for  $w_F$  (increase) but in opposite directions for  $w_M$ . Aggregate labor supply contracts, putting upward pressure on both  $w_F$  and  $w_M$ , while relative market effects put upward pressure on  $w_F$  and downward pressure on  $w_M$ .

If  $M$  and  $F$  have the same reservation wage cdf, then  $S_F \leq S_M$  because we always have  $w_F \leq w_M$ . If the two groups have different cdfs for their reservation wages, then  $S_F \geq S_M$  is possible. For example, it is likely that the group that suffers discrimination has lower assets, which could imply that at any given wage the  $F$  employment rate is higher than the  $M$  employment rate. Then we will have  $S_F > S_M$  if the effect of lower assets is greater than the effect of lower wages.

Table 1 summarizes the direction of the effects on wage levels. The signs of the preference distribution effect reverse for  $d'_F(p_M) \geq 0$ . The rows indicate the conditions necessary to sign the effects. If discrimination preferences are fixed, then the preference distribution effect is zero. In that case relative employment rates give sufficient information to sign the aggregate market effect, and if aggregate and relative market effects have the same sign, then the overall effect can also be signed.

If discrimination preferences vary with population composition, then signing the overall effect requires considerably more information.

Table 1

Effects of changing population composition on wage levels

	$\frac{dw_F}{dp_M}$	$\frac{dw_M}{dp_M}$
<i>Relative market effect</i>	+	–
<i>Aggregate market effect</i> (Present only with variable labor supply)		
$S_M(w_M) < S_F(w_F)$	+	+
$S_M(w_M) > S_F(w_F)$	–	–
$S_M(w_M) = S_F(w_F)$	0	0
<i>Preference distribution effect, with <math>d'_F(p_M) \leq 0</math></i> (Present only when discrimination preferences vary with population composition )		
$\frac{E_1 - p_M S'_M}{E_3 w_F / w_M^2} - 1 < / > \frac{D_1}{D_2}$	+ / –	
$\frac{E_2 - p_F S'_F}{E_3 / w_M} - 1 > / < \frac{D_2}{D_1}$		+ / –

We can measure the responsiveness of wages to changes in population

composition using the population composition elasticity of wages,  $\eta_{KH} = \frac{dw_K}{dp_H} \cdot \frac{p_H}{w_K}$ ,

for  $K, H = M, F$ . The sign of these elasticities is the same as the sign of  $\frac{dw_K}{dp_H}$ . Their

magnitude varies with the elasticities of labor supply,  $\varepsilon_K = S'_K(w_K) \frac{w_K}{S_K}$ ,  $K = M, F$ .

Population composition elasticities are inversely related to own group elasticity of labor supply. For example, if  $\varepsilon_K$  increases, then  $|\eta_{KH}|$  decreases, since the larger the labor supply adjustment to a change in population composition, the smaller the wage adjustment. This is not necessarily the case with cross group effects of labor supply elasticities, where the direction of the effects varies with the sign of both  $\frac{dw_M}{dp_K}$  and

$$\frac{dw_F}{dp_K}.$$

The direction of the effects of changing population composition on the wage ratio is also uncertain. Substituting  $\frac{dw_F}{dp_M}$  and  $\frac{dw_M}{dp_M}$  into  $\frac{d}{dp_M}(\frac{w_F}{w_M})$  we obtain:

$$\begin{aligned} \frac{d}{dp_M}(\frac{w_F}{w_M}) &= \frac{1}{w_M \Delta} \cdot \\ &\cdot [S_F(E_1 - p_M S'_M) + \frac{w_F}{w_M} S_M(E_2 - p_F S'_F) - \frac{w_F}{w_M} (E_2 - p_F S'_F) D_1 + (E_1 - p_M S'_M) D_2] \end{aligned} \quad (9)$$

Here we have only a relative market and a preference distribution effect. The direction of the relative market effect is to increase the  $F/M$  wage ratio when  $p_M$  increases while the direction of the preference distribution effect depends on whether  $d'_F(p_M) \leq 0$  or  $d'_F(p_M) \geq 0$ . If increasing  $p_M$  results in less discriminatory preferences, i.e.

$d'_F(p_M) \leq 0$ , then the relative market and distributional effects have the same

direction. If however the conditions of the inter-group contact theory apply, then the

two effects work in opposite directions and we cannot determine a priori which dominates.

If at the initial equilibrium we have  $d_F > 0$ , it is possible that changes in population composition result in an equilibrium with  $d_F = 0$ . This can occur with

$d'_F(p_M) \leq 0$  if the original equilibrium wage ratio is sufficiently close to  $\frac{w_F}{w_M} = 1$  and

$p_M$  increases, or with  $d'_F(p_M) \geq 0$  if  $p_M$  decreases.

The research on wage differentials between white and black men in the US has found that productivity differentials account for a substantial part of their wage gap (Neal and Johnson, 1996, Bowlus and Eckstein, 2002). The results above can be generalized to allow for productivity differences between the two population groups. Aggregate and relative market effects are still present when the productivity of  $F$  workers is only a fraction  $\alpha$  of that of the  $M$  workers, where  $0 < \alpha < 1$ , and the production function takes the form  $Q = f(L_M + \alpha L_F)$ . However, the sign of the aggregate effect now depends on the relative magnitudes of  $S_M(w_M)$  and  $\alpha S_F(w_F)$  instead of  $S_F(w_F)$ . That is, with productivity differences it is the *effective* employment rates of the two groups that matter. In this formulation units of labor of the  $F$  workers are converted into units of labor of the  $M$  workers in order to compare the effective employment rates of the two groups.

#### 4. Conclusion

In the context of employer discrimination with variable labor supplies and discrimination preferences varying with population composition, a changing population composition has three effects on wage *levels*: (i) a relative markets effect which is always negative for the group whose proportion increases in the population

and is always present when population composition varies, whether labor supply and/or discrimination preferences vary or not; (ii) an aggregate market effect, the direction of which depends on the relative employment rates of the two groups and is present only with variable labor supply; and (iii) a preference distribution effect the direction of which depends on the way distributional shifts affect the demands for the two groups, and is present only when preferences vary. These effects may work in opposite directions, in which case the direction of the overall effect depends on their relative magnitudes. If discrimination preferences are fixed then relative employment rates give sufficient information to sign the aggregate market effect, and if aggregate and relative market effects have the same sign, then the overall effect can be signed as well. If discrimination preferences vary with population composition, then signing the overall effect requires considerably more information. The direction of the effect on the wage *ratio* is more predictable. If discrimination increases as the proportion of the discriminated group increases, relative market and distributional effects work in the same direction to increase the relative wage differential. If discrimination decreases as the proportion of the discriminated group increases, then relative market and distributional effects work in opposite directions and the effect on wage ratio depends on their relative magnitudes.

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## Appendix

### I. Wage effects with variable preferences

Result: The effects of a changing population composition on equilibrium wages are given by:

$$\frac{dw_F}{dp_M} = \Delta^{-1} \cdot$$

$$[E_3 \frac{w_F}{w_M^2} (S_M - S_F) + S_F (E_1 - p_M S'_M) - E_3 \frac{w_F}{w_M^2} D_1 + (E_1 - p_M S'_M - E_3 \frac{w_F}{w_M^2}) D_2]$$

$$\frac{dw_M}{dp_M} = \Delta^{-1} \cdot$$

$$[\frac{E_3}{w_M} (S_M - S_F) - S_M (E_2 - p_F S'_F) + (E_2 - p_F S'_F - \frac{E_3}{w_M}) D_1 - \frac{E_3}{w_M} D_2]$$

where

$$E_1 = \frac{\partial}{\partial w_M} [ \int_0^{w_F/w_M} R(w_M) g(x; p_M) dx ] = R'(w_M) \cdot \int_0^{w_F/w_M} g(x; p_M) dx < 0$$

$$E_2 = \frac{\partial}{\partial w_F} [ \int_{w_F/w_M}^1 R(\frac{w_F}{x}) \frac{1}{x} g(x; p_M) dx ] = \int_{w_F/w_M}^1 R'(\frac{w_F}{x}) \frac{1}{x} g(x; p_M) dx < 0$$

$$E_3 = R(w_M) g(\frac{w_F}{w_M}; p_M) > 0$$

$$D_1 = R(w_M) \int_0^{w_F/w_M} g_M(x; p_M) dx = \frac{\partial}{\partial p_M} [ \int_0^{w_F/w_M} R(w_M) g(x; p_M) dx ]$$

$$D_2 = \int_{w_F/w_M}^1 R(\frac{w_F}{x}) g_M(x; p_M) dx = \frac{\partial}{\partial p_M} [ \int_{w_F/w_M}^1 R(\frac{w_F}{x}) g(x; p_M) dx ]$$

$$\Delta = -(E_1 - p_M S'_M)(E_2 - p_F S'_F - \frac{E_3}{w_M}) + (E_2 - p_F S'_F) E_3 \frac{w_F}{w_M^2} < 0$$

$$p_F = 1 - p_M$$

### Derivation

Differentiating (6) in the text wrt to  $p_M$ , the LHS is unaffected by the dependence of discrimination preferences on  $p_M$ :

$$\frac{d}{dp_M}[p_M S_M(w_M)] = S_M(w_M) + p_M S'_M(w_M) \frac{dw_M}{dp_M} \quad (\text{A I.1})$$

The RHS gives:

$$\begin{aligned} \frac{d}{dp_M} \left[ \int_0^{w_F/w_M} R(w_M) g(x; p_M) dx \right] &= \frac{d}{dp_M} \left[ R(w_M) \int_0^{w_F/w_M} g(x; p_M) dx \right] = \\ &= R'(w_M) \frac{dw_M}{dp_M} \int_0^{w_F/w_M} g(x; p_M) dx + \\ &+ R(w_M) \left[ g\left(\frac{w_F}{w_M}; p_M\right) \left( \frac{1}{w_M} \cdot \frac{dw_F}{dp_M} - \frac{w_F}{w_M^2} \frac{dw_M}{dp_M} \right) + \int_0^{w_F/w_M} g_M(x; p_M) dx \right] \end{aligned} \quad (\text{A I.2})$$

The term due to the preferences being a function of the population composition here is:

$$R(w_M) \int_0^{w_F/w_M} g_M(x; p_M) dx$$

The sign of this term depends on how the preference distribution changes as  $p_M$  changes. If  $d'_F(p_M) \leq 0$  for all  $p_M$ , then because  $g(x; p'_M)$  first order stochastically dominates  $g(x; p''_M)$  for  $p' \geq p''$ , we have:

$$\int_0^{w_F/w_M} g_M(x; p_M) dx \leq 0.$$

This is because:

$$\int_0^{w_F/w_M} g_M(x; p_M) dx = \frac{d}{dp_M} \left[ \int_0^{w_F/w_M} g(x; p_M) dx \right]$$

and by first-order stochastic dominance we have:

$$\int_0^{w_F/w_M} g(x; p'_M) dx \leq \int_0^{w_F/w_M} g(x; p''_M) dx$$



If  $d'_F(p_M) \geq 0$  for all  $p_M$ , then because  $g(x; p''_M)$  first order stochastically dominates

$g(x; p'_M)$  for  $p' \geq p''_M$ , we have:

$$\int_0^{w_F/w_M} g_M(x; p_M) dx \geq 0$$

If  $d'_F(p_M)$  is not monotonic, then  $\int_0^{w_F/w_M} g_M(x; p_M) dx$  cannot be readily signed.

Turning to (7) and differentiating wrt to  $p_M$ , the LHS is again unaffected by the dependence of discrimination preferences on population composition:

$$\frac{d}{dp_M}[(1-p_M)S_F(w_F)] = -S_F(w_F) + (1-p_M)S'_F(w_F) \frac{dw_F}{dp_M} \quad (\text{A I.3})$$

From the RHS we have:

$$\begin{aligned} \frac{d}{dp_M} \left[ \int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g(x; p_M) dx \right] &= \\ &= -\frac{d}{dp_M} \left( \frac{w_F}{w_M} \right) \cdot R\left(\frac{w_F}{w_F/w_M}\right) g\left(\frac{w_F}{w_M}; p_M\right) + \int_{w_F/w_M}^1 \frac{\partial}{\partial p_M} \left[ R\left(\frac{w_F}{x}\right) g(x; p_M) \right] dx = \\ &= -R(w_M) g\left(\frac{w_F}{w_M}; p_M\right) \left( \frac{1}{w_M} \cdot \frac{dw_F}{dp_M} - \frac{w_F}{w_M^2} \frac{dw_M}{dp_M} \right) + \\ &+ \int_{w_F/w_M}^1 R'\left(\frac{w_F}{x}\right) \frac{1}{x} g(x; p_M) dx \frac{dw_F}{dp_M} + \int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p_M) dx \end{aligned} \quad (\text{A I.4})$$

The additional term due to preferences being a function of population composition is:

$$\int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p_M) dx$$

The sign of this term depends on how the preference distribution changes as  $p_M$

changes. If  $d'_F(p_M) \leq 0$  for all  $p_M$ , then because  $g(x; p'_M)$  first-order stochastically

dominates  $g(x; p''_M)$  for every  $p' \geq p''_M$ , we have:

$$\int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p_M) dx \geq 0$$

This is because if  $g(x; p'_M)$  first-order stochastically dominates  $g(x; p''_M)$  for every

$p' \geq p''_M$ , this is equivalent to:

$$\int s(x) g_M(x; p'_M) dx \geq \int s(x) g_M(x; p''_M) dx$$

for every nondecreasing function  $s(x)$ . Then let:

$$s(x) = \begin{cases} 0 & \text{for } x < \frac{w_F}{w_M} \\ R\left(\frac{w_F}{x}\right) & \text{for } x \geq \frac{w_F}{w_M} \end{cases}$$

Then

$$\int_{-\infty}^1 s(x) g(x; p_M) dx = \int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g(x; p_M) dx$$

and by the first-order stochastic dominance

$$\int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p'_M) dx \geq \int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p''_M) dx$$

Similarly, if  $d'_F(p_M) \geq 0$  for all  $p_M$ , then because  $g(x; p''_M)$  first order stochastically

dominates  $g(x; p'_M)$  for  $p' \geq p''_M$ , we have:

$$\int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p_M) dx \leq 0$$

If  $d_F(p_M)$  is not monotonic  $\int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p_M) dx$  cannot be readily signed.

Now letting

$$R(w_M) \int_0^{w_F/w_M} g_M(x; p_M) dx = D_1$$

we solve (A I.1)-(A I.2) for  $\frac{dw_M}{dp_M}$  to obtain:

$$\frac{dw_M}{dp_M} = \frac{S_M(w_M) - R(w_M)g\left(\frac{w_F}{w_M}; p_M\right) \frac{1}{w_M} \frac{dw_F}{dp_M} - D_1}{R'(w_M) \int_0^{w_F/w_M} g(x; p_M) dx - p_M S'_M(w_M) - R(w_M)g\left(\frac{w_F}{w_M}; p_M\right) \frac{w_F}{w_M^2}} \quad (\text{A I.5})$$

Letting:

$$\int_{w_F/w_M}^1 R\left(\frac{w_F}{x}\right) g_M(x; p_M) dx = D_2$$

we solve (A I.3)-(A I.4) for  $\frac{dw_M}{dp_M}$  to obtain:

$$\frac{dw_M}{dp_M} = \frac{-S_F(w_F) + [(1 - p_M)S'_F(w_F) - \int_{w_F/w_M}^1 R'\left(\frac{w_F}{x}\right) \frac{1}{x} g(x; p_M) dx + R(w_M)g\left(\frac{w_F}{w_M}; p_M\right) \frac{1}{w_M}] \cdot \frac{dw_F}{dp_M} - D_2}{R(w_M)g\left(\frac{w_F}{w_M}; p_M\right) \frac{w_F}{w_M^2}} \quad (\text{A I.6})$$

Using:

$$E_1 = R'(w_M) \cdot \int_0^{w_F/w_M} g(x; p_M) dx < 0$$

$$E_2 = \int_{w_F/w_M}^1 R'\left(\frac{w_F}{x}\right) \frac{1}{x} g(x; p_M) dx < 0$$

$$E_3 = R(w_M)g\left(\frac{w_F}{w_M}; p_M\right) > 0$$

and simplifying notation, from (A I.5) we have:

$$\frac{dw_M}{dp_M} = \frac{S_M - \frac{E_3}{w_M} \frac{dw_F}{dp_M} - D_1}{E_1 - p_M S'_M - E_3 \frac{w_F}{w_M^2}} \quad (\text{A I.5'})$$

From (A I.6) we have:

$$\frac{dw_M}{dp_M} = \frac{-S_F + (p_F S'_F - E_2 + \frac{E_3}{w_M}) \cdot \frac{dw_F}{dp_M} - D_2}{E_3 \frac{w_F}{w_M^2}} \quad (\text{A I.6'})$$

Solving for  $\frac{dw_F}{dp_M}$  we find (8.1) and substituting in (A I.6') we find (8.2).

## II. Fixed preferences and productivity differences

Suppose the two types of workers,  $M$  and  $F$ , have different productive characteristics. Without loss of generality assume that  $F$  is less productive than  $M$ . Suppose the production function takes the form:

$$Q = f(L_M + \alpha L_F) \quad \text{with } 0 < \alpha < 1$$

This formulation implies that the marginal product of an  $F$  worker is a fraction  $\alpha$  of the marginal product of an  $M$  worker. Without wage discrimination and competitive markets we would expect the  $F/M$  wage ratio to be  $\alpha$ . With employer discrimination preferences, the first order conditions become:

$$f' = w_M \quad \text{if } L_M > 0, \text{ and}$$

$$f' < w_M \quad \text{if } L_M = 0$$

$$\alpha f' = w_F (1 + d_F) \Rightarrow f' = w_F \frac{(1 + d_F)}{\alpha} \quad \text{if } L_F > 0, \text{ and}$$

$$f' < w_F \frac{(1 + d_F)}{\alpha} \quad \text{if } L_F = 0$$

For a given wage ratio, firms with  $d_F$  such that  $\alpha w_M > w_F (1 + d_F)$  hire only  $F$  workers and firms with  $d_F$  such that  $\alpha w_M < w_F (1 + d_F)$  hire only  $M$  workers. With  $x$  defined as

$x = \frac{1}{1 + d_F}$  and with fixed discrimination preferences, the equilibrium wage ratio is

determined by:

$$p_M S_M(w_M) = \int_0^{w_F / \alpha w_M} R(w_M) g(x) dx \quad (\text{A II.1})$$

$$p_F \alpha S_F(w_F) = \int_{w_F / \alpha w_M}^1 R\left(\frac{w_F}{\alpha x}\right) g(x) dx \quad (\text{A II.2})$$

Totally differentiating (A II.1) and solving for  $\frac{dw_M}{dp_M}$  we have:

$$\begin{aligned} \frac{d}{dp_M} [p_M S_M(w_M)] &= \frac{d}{dp_M} \left[ \int_0^{w_F / \alpha w_M} R(w_M) g(x) dx \right] \Rightarrow \\ &\Rightarrow S_M(w_M) + p_M S'_M(w_M) \frac{dw_M}{dp_M} = \\ &= R'(w_M) \frac{dw_M}{dp_M} \cdot \int_0^{w_F / \alpha w_M} g(x) dx + R(w_M) g\left(\frac{w_F}{\alpha w_M}\right) \left[ \frac{1}{\alpha w_M} \cdot \frac{dw_F}{dp_M} - \frac{w_F}{\alpha w_M^2} \cdot \frac{dw_M}{dp_M} \right] \Rightarrow \\ &\Rightarrow \frac{dw_M}{dp_M} = \frac{S_M(w_M) - R(w_M) g\left(\frac{w_F}{\alpha w_M}\right) \cdot \frac{1}{\alpha w_M} \cdot \frac{dw_F}{dp_M}}{R'(w_M) \cdot \int_0^{w_F / \alpha w_M} g(x) dx - p_M S'_M(w_M) - R(w_M) g\left(\frac{w_F}{\alpha w_M}\right) \frac{w_F}{\alpha w_M^2}} \quad (\text{A II.3}) \end{aligned}$$

Totally differentiating (A II.2) we have:

$$\begin{aligned} \frac{d}{dp_M} [(1 - p_M) \alpha S_F(w_F)] &= \frac{d}{dp_M} \left[ \int_{w_F / \alpha w_M}^1 R\left(\frac{w_F}{\alpha x}\right) g(x) dx \right] \Rightarrow \\ &\Rightarrow -\alpha S_F(w_F) + (1 - p_M) \alpha S'_F(w_F) \frac{dw_F}{dp_M} = \\ &= \left[ \int_{w_F / \alpha w_M}^1 R'\left(\frac{w_F}{\alpha x}\right) \frac{1}{\alpha x} g(x) dx \right] \frac{dw_F}{dp_M} - R(w_M) g\left(\frac{w_F}{\alpha w_M}\right) \left[ \frac{1}{\alpha w_M} \cdot \frac{dw_F}{dp_M} - \frac{w_F}{\alpha w_M^2} \cdot \frac{dw_M}{dp_M} \right] \Rightarrow \\ &\Rightarrow \frac{dw_M}{dp_M} = \\ &= \frac{-\alpha S_F(w_F) + \frac{dw_F}{dp_M} [(1 - p_M) \alpha S'_F(w_F) - \int_{w_F / \alpha w_M}^1 R'\left(\frac{w_F}{\alpha x}\right) \frac{1}{\alpha x} g(x) dx + R(w_M) g\left(\frac{w_F}{\alpha w_M}\right) \frac{1}{\alpha w_M}]}{R(w_M) g\left(\frac{w_F}{\alpha w_M}\right) \frac{w_F}{\alpha w_M^2}} \quad (\text{A II.4}) \end{aligned}$$

Solving for  $\frac{dw_F}{dp_M}$  we obtain:

$$\frac{dw_F}{dp_M} = \Delta_\alpha^{-1} [E_3 \frac{w_F}{\alpha w_M^2} [S_M(w_M) - \alpha S_F(w_F)] + \alpha S_F(w_F) [E_1 - p_M S'_M(w_M)]] \quad (\text{A II.5})$$

where:

$$E_{\alpha 1} = \frac{\partial}{\partial w_M} \left[ \int_0^{w_F / \alpha w_M} R(w_M) g(x) dx \right] = R'(w_M) \cdot \int_0^{w_F / \alpha w_M} g(x) dx < 0$$

$$E_{\alpha 2} = \frac{\partial}{\partial w_F} \left[ \int_{w_F / \alpha w_M}^1 R\left(\frac{w_F}{\alpha x}\right) \frac{1}{\alpha x} g(x) dx \right] = \int_{w_F / \alpha w_M}^1 R'\left(\frac{w_F}{\alpha x}\right) \frac{1}{\alpha x} g(x) dx < 0$$

$$E_{\alpha 3} = R(w_M) g\left(\frac{w_F}{\alpha w_M}\right) > 0$$

$$\Delta_\alpha = -(E_1 - p_M S'_M)(E_2 - p_F \alpha S'_F - \frac{E_3}{\alpha w_M}) + (E_2 - p_F \alpha S'_F) E_3 \frac{w_F}{\alpha w_M^2} < 0$$

We have shown that with  $\alpha = 1$ , if  $S_M > S_F$  then  $\frac{dw_F}{dp_M} < 0$  is possible. With

$0 < \alpha < 1$ , it is still possible to have  $\frac{dw_F}{dp_M} < 0$  if  $S_M > \alpha S_F$ . The direction of the

aggregate effect is now determined by the productivity adjusted employment rates.

With productivity differences it is possible for an increase in  $p_M$  to reduce  $w_F$  even

when  $S_M < S_F$ , as long as  $S_M > \alpha S_F$ . This is because with the increase in  $p_M$

aggregate supply increases because the  $M$  are relatively more productive.

Substituting for  $\frac{dw_F}{dp_M}$  in (A II.4) we obtain:

$$\frac{dw_M}{dp_M} = \Delta_\alpha^{-1} \left[ \frac{E_{\alpha 3}}{\alpha w_M} (S_M - \alpha S_F) - S_M (E_2 - p_F \alpha S'_F) \right] \quad (\text{A II.6})$$

We have shown that  $\frac{dw_M}{dp_M} > 0$  is possible with  $\alpha = 1$ , if  $S_M < S_F$ . With  $0 < \alpha < 1$  it is

still possible to have  $\frac{dw_M}{dp_M} > 0$  if  $S_M < \alpha S_F$ . The effect  $\frac{dw_M}{dp_M} > 0$  requires that the

aggregate labor supply effect of an increase in  $p_M$  is positive, that is, that aggregate

labor supply shifts to the left. For this to happen with  $0 < \alpha < 1$  it is no longer sufficient to have  $S_M < S_F$ . This is because an increase in  $p_M$  can still increase aggregate labor supply because the  $M$  are more productive. So the employment rate of the  $M$  must be sufficiently lower than the employment rate of  $F$  in order to generate the shift of the aggregate labor supply to the left.

### III. Population composition elasticities and labor supply elasticities

To express equations (8.1) and (8.2) in terms of labor supply elasticities, define the wage elasticity of supply of group  $G$ :

$$\varepsilon_G = S'_G(w_G) \frac{w_G}{S_G} = \frac{\partial}{\partial w_G} \left( \frac{L_G}{P_G} \right) \cdot \frac{w_G}{L_G / P_G} = \frac{1}{P_G} \cdot \frac{\partial L_G}{\partial w_G} \cdot P_G \cdot \frac{w_G}{L_G} = \frac{\partial L_G}{\partial w_G} \cdot \frac{w_G}{L_G}$$

where  $P_G$  is the population of group  $G$ ,  $P$  is the total population, and  $L_G$  is the headcount of employed workers of group  $G$ , for  $G=F, M$ .

This elasticity measures the percentage change in the employment rate of group  $G$  in response to a percentage change in the wages of group  $G$ . Substituting into (8.1) and (8.2) we obtain:

$$\frac{dw_F}{dp_M} = \frac{E_3 \frac{w_F}{w_M^2} (S_M - S_F) + S_F (E_1 - \frac{p_M}{w_M} \varepsilon_M S_M)}{(\frac{p_M}{w_M} \varepsilon_M S_M - E_1)(E_2 - \frac{E_3}{w_M} - \frac{p_F}{w_F} \varepsilon_F S_F) - (\frac{p_F}{w_F} \varepsilon_F S_F - E_2) E_3 \frac{w_F}{w_M^2}} \quad (\text{A III.1})$$

and

$$\frac{dw_M}{dp_M} = \frac{\frac{E_3}{w_M} (S_M - S_F) + S_M (\frac{p_F}{w_F} \varepsilon_F S_F - E_2)}{(\frac{p_M}{w_M} \varepsilon_M S_M - E_1)(E_2 - \frac{E_3}{w_M} - \frac{p_F}{w_F} \varepsilon_F S_F) - (\frac{p_F}{w_F} \varepsilon_F S_F - E_2) E_3 \frac{w_F}{w_M^2}} \quad (\text{A III.2})$$

The population composition elasticity of  $F$  wages is given by:

$$\begin{aligned}
\eta_{FM} &= \frac{dw_F}{dp_M} \cdot \frac{p_M}{w_F} = \\
&= \frac{\frac{E_3}{w_M^2}(S_M - S_F) + \frac{S_F}{w_F}(E_1 - \frac{p_M}{w_M}\varepsilon_M S_M)}{(\frac{S_M}{w_M}\varepsilon_M - \frac{E_1}{p_M})(E_2 - \frac{E_3}{w_M} - \frac{p_F}{w_F}\varepsilon_F S_F) - (\frac{p_F}{w_F}\varepsilon_F S_F - E_2)\frac{E_3}{p_M} \cdot \frac{w_F}{w_M^2}}
\end{aligned} \tag{A III.3}$$

The population composition elasticity of  $M$  wages is given by:

$$\begin{aligned}
\eta_{MM} &= \frac{dw_M}{dp_M} \cdot \frac{p_M}{w_M} = \\
&= \frac{\frac{E_3}{w_M}(S_M - S_F) + S_M(\frac{p_F}{w_F}\varepsilon_F S_F - E_2)}{(\varepsilon_M S_M - E_1\frac{w_M}{p_M})(E_2 - \frac{E_3}{w_M} - \frac{p_F}{w_F}\varepsilon_F S_F) - (\frac{p_F}{w_F}\varepsilon_F S_F - E_2)\frac{E_3}{p_M} \cdot \frac{w_F}{w_M}}
\end{aligned} \tag{A III.4}$$

The population composition elasticities of wages in terms of  $p_F$  ( $p_F = 1 - p_M$ ) are derived similarly. Each of the population composition elasticities depends on the elasticities of supply of both groups. Differentiating  $\eta_{FM}$  wrt  $\varepsilon_F$  we obtain:

$$\begin{aligned}
\frac{\partial \eta_{FM}}{\partial \varepsilon_F} &= \frac{\frac{E_3}{w_M^2}(S_M - S_F) + \frac{S_F}{w_F}(E_1 - \frac{p_M}{w_M}\varepsilon_M S_M)}{[(\frac{S_M}{w_M}\varepsilon_M - \frac{E_1}{p_M})(E_2 - \frac{E_3}{w_M} - \frac{p_F}{w_F}\varepsilon_F S_F) - (\frac{p_F}{w_F}\varepsilon_F S_F - E_2)\frac{E_3}{p_M} \cdot \frac{w_F}{w_M^2}]^2} \\
&\quad \cdot \frac{p_F}{w_F} S_F (\frac{S_M}{w_M}\varepsilon_M - \frac{E_1}{p_M} + \frac{E_3}{p_M} \cdot \frac{w_F}{w_M^2})
\end{aligned} \tag{A III.5}$$

Since  $E_1 < 0$ ,  $E_3 > 0$ , and  $\varepsilon_M > 0$ ,  $\frac{\partial \eta_{FM}}{\partial \varepsilon_F}$  has the opposite sign of  $\frac{dw_F}{dp_M}$  and its sign

varies for the same reasons the sign of  $\frac{dw_F}{dp_M}$  varies, i.e. depending on the relative

employment rates of the two groups and on the relative sizes of the aggregate market

and relative market effects. For example, if  $S_M < S_F$  and  $p_M$  increases, we have

already seen that  $w_F$  unambiguously increases, as both aggregate and relative market



effects work in the same direction. This implies that  $\frac{\partial \eta_{FM}}{\partial \varepsilon_F} < 0$ , i.e. the greater the elasticity of the  $F$  supply, the smaller the increase in  $w_F$ . This is because increases in  $w_F$  induce larger labor supply responses (expansions in this case) and therefore  $w_F$  does not need to increase as much to adjust to the change in population composition. If on the other hand  $S_M > S_F$  and  $p_M$  increases, if the aggregate market effect is

greater than the relative market effect,  $w_F$  will decrease. This implies that  $\frac{\partial \eta_{FM}}{\partial \varepsilon_F} > 0$ .

In this case  $\eta_{FM}$  is negative and therefore a positive sign for  $\frac{\partial \eta_{FM}}{\partial \varepsilon_F}$  means that  $\eta_{FM}$  increases algebraically and hence the elasticity decreases. The reason here is the same as previously: larger labor supply responses from the  $F$  (contractions in this case) reduce the adjustment required from the wage. In sum, regardless of whether  $\eta_{FM}$  is positive or negative, the greater  $\varepsilon_F$ , the smaller the population composition elasticity of  $w_F$ .

Turning to the relationship between  $\eta_{FM}$  and  $\varepsilon_M$  we have:

$$\frac{\partial \eta_{FM}}{\partial \varepsilon_M} = \frac{S_M \varepsilon_M}{(w_M)^3} \cdot E_3 \left[ \frac{E_3}{w_M} (S_M - S_F) + S_M \left( \frac{S_F}{w_F} p_F \varepsilon_F - E_2 \right) \right] /$$

$$\left[ \left( \frac{S_M}{w_M} \varepsilon_M - \frac{E_1}{p_M} \right) \left( E_2 - \frac{E_3}{w_M} - \frac{p_F}{w_F} \varepsilon_F S_F \right) - \left( \frac{p_F}{w_F} \varepsilon_F S_F - E_2 \right) \frac{E_3}{p_M} \cdot \frac{w_F}{w_M^2} \right]^2$$

(A III.6)

The sign of  $\frac{\partial \eta_{FM}}{\partial \varepsilon_M}$  is the opposite of  $\frac{dw_M}{dp_M}$ . The relationship between  $\eta_{FM}$  and  $\varepsilon_M$  is

more complicated than the relationship between  $\eta_{FM}$  and  $\varepsilon_F$  where  $\frac{\partial \eta_{FM}}{\partial \varepsilon_F}$  varied

with the sign of  $\frac{dw_F}{dp_M}$ . This is because while  $\frac{\partial \eta_{FM}}{\partial \varepsilon_M}$  has the opposite sign of  $\frac{dw_M}{dp_M}$ ,

$\eta_{FM}$  has the same sign as  $\frac{dw_F}{dp_M}$ .

Consider for example the case when  $S_M > S_F$  which unambiguously implies

$\frac{\partial \eta_{FM}}{\partial \varepsilon_M} > 0$ . This is the case where as  $p_M$  increases both the aggregate and the relative

market effects work in the direction of decreasing  $w_M$  (i.e. we unambiguously

have  $\frac{dw_M}{dp_M} < 0$ .) Algebraically  $\eta_{FM}$  increases as  $\varepsilon_M$  increases. But in this case, the  $F$

wage may increase or decrease depending on the size of the aggregate and relative

market effects. So  $\eta_{FM}$  may be positive if the relative market effect dominates the

aggregate market effect or negative if the opposite is the case. If  $\eta_{FM}$  is positive, then

elasticity increases and the increase in  $F$  wage is greater. If  $\eta_{FM}$  is negative elasticity

decreases, and there is a lesser decline in  $F$  wages.

Similarly  $\frac{\partial \eta_{MM}}{\partial \varepsilon_M}$  has the opposite sign of  $\frac{dw_M}{dp_M}$ , and  $\frac{\partial \eta_{MM}}{\partial \varepsilon_F}$  has the opposite

sign of  $\frac{dw_F}{dp_M}$ . Therefore as  $\varepsilon_G$  increases, the *own* population composition elasticity of

wages  $|\eta_{GH}|$  decreases,  $G, H = M, F$ . The cross elasticities however may either

increase or decrease as  $\varepsilon_G$  increases.